Math 10B with Professor Stankova
Quiz 6; Tuesday, 3/5/2019
Section #203; Time: 11 AM
GSI name: Roy Zhao **Name:** _____

Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. True **FALSE** To show that X, Y are not independent random variables, we need to show that $P(X = x, Y = y) \neq P(X = x)P(Y = y)$ for all choices of x, y.

> **Solution:** We only need to show it for a single counterexample of x, y.

2. True **FALSE** If x is not in the range of X and f is the PMF of X, then $f(x)$ does not exist.

> **Solution:** $f(x) = P(X = x)$ and since x is not in the range, then $f(x) = P(X = x) = 0$. So it is defined but equal to 0.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (6 points) I am playing a game where I flip a coin over and over until I either flip a tails, or flip the coin 4 times. Let X be the random variable for how many times I need to flip the coin. Compute and draw the PMF of X. (Hint: Can you flip the coin 5 times? Calculate the range of X first)

> **Solution:** The game must end by the end of the 4rd round so the range of X is $\{1, 2, 3, 4\}$. Then $P(X = 1) = \frac{1}{2}$ because the only way it ends is if we flip a tails. Then $P(X = 2) = \frac{1}{2} \cdot \frac{1}{2}$ because we need to first flip a heads then flip a tail. Similarly $P(X = 3) = \frac{1}{2^3}$ because the only way this can happen is HHT. Finally, we have that $P(X = 4) = 1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{8} = \frac{1}{8}$ because that is if the game does not end in the first, second, or third round. So the PMF is
>
> | x | 1 | 2 | 3 | 4 |
> |---|---|---|---|---|
> | $f(x)$ | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |

(b) (2 points) Let Y be the random variable that is 1 if the first flip is a tails and 0 otherwise. What is the PMF of Y?

Solution:

x	0	1
$f(x)$	$\frac{1}{2}$	$\frac{1}{2}$

(c) (2 points) Are X and Y independent random variables?

Solution: No they are not. Intuitively if we know that $Y = 1$, then we know that we flipped a tails so we know that the game ended and so $X = 1$. In math, this says that

$$P(X = 1, Y = 1) = \frac{1}{2} \neq P(X = 1)P(Y = 1) = \frac{1}{4}.$$